$C_{el} = \frac{1}{2}\pi^2 k_B N(\frac{T}{T_f})$, which is the expression for electronic heat capacity.

ELECTRONIC THERMAL CONDUCTIVITY OF A METAL (K_{el})

Thermal conductivity is normally greater in metals than in non-metals. Thermal energy is a result of transfer of electrons. It is known that electrons coming from hotter regions of the metal carry more thermal energy than those from cooler regions of the metal, thus resulting in a net flow of heat. Using elementary kinetic theory, the electronic thermal conductivity is given by;

$K_{el} = \frac{1}{3}C_{el}v_f l$, where v_f is the mean speed of electrons responsible for thermal conductivity, C_{el} is the electronic heat capacity per unit volume, l is the mean free path.

The mean free path of the electrons is given by $l = v_f\tau$, where τ is the relaxation time or time of collision of electrons.

Assume a fermi gas,

$\frac{1}{2}mv_f^2 = E_f$

$v_f = (\frac{2E_f}{m})^{\frac{1}{2}}$.

Hence $K_{el} = \frac{1}{3}C_{el}(\frac{2E_f}{m})^{\frac{1}{2}}v_f\tau$.

$K_{el} = \frac{1}{3}C_{el}(\frac{2E_f}{m})\tau$.

Using $C_{el} = \frac{1}{2}\pi^2 Nk_B(\frac{T}{T_f})$ and $E_f = k_BT_f$,

$K_{el} = \frac{1}{3}.\frac{1}{2}\pi^2 Nk_B(\frac{T}{T_f}).(\frac{2k_BT_f}{m})\tau$.

$K_{el} = \frac{1}{3}\pi^2\frac{Nk_B^2\tau}{m}T$.

$K_{el} \propto T$, where the constant is given by $\frac{1}{3}\pi^2\frac{Nk_B^2\tau}{m}$.

WIEDEMANN-FRANZ LAW

It states that for metals at not low temperatures, the ratio of thermal conductivity to electrical conductivity is directly proportional to the temperature with the value of the constant of proportionality being independent of a particular metal.

Recall that $\sigma = \frac{Ne^2\tau}{m}$ and $K_{el} = (\frac{1}{3}\pi^2 \frac{Nk_B^2\tau}{m})T$.

$\frac{K_{el}}{\sigma} = \frac{1}{3}\pi^2 \frac{Nk_B^2\tau}{m} T . \frac{m}{Ne^2\tau}$.

$\frac{K_{el}}{\sigma} = \frac{\pi^2}{3}(\frac{k_B}{e})^2 T$, which is Wiedemann Franz law.

Thus dividing by T gives $L = \frac{K_{el}}{T\sigma} = \frac{\pi^2}{3}(\frac{k_B}{e})^2$, which is the Lorentz number.

ELECTRICAL CONDUCTIVITY OF METALS

In most metals electrical resistivity ($\rho = \frac{1}{\tau}$) is dominated at room temperature i.e 300 K, by collisions of the conduction electrons with lattice phonons and at liquid helium temperature which is about 4 K by collisions with impurity atoms and mechanical imperfections in the lattice. In these colllisions the net relaxation rate is given by $\frac{1}{\tau} = \frac{1}{\tau_L} + \frac{1}{\tau_i}$, where τ_L and τ_i are the collision times for scattering by phonons and imperfections respectively.

The net resistivity is thus given by $\rho = \rho_L + \rho_i$, where ρ_L is the resistivity caused by thermal phonons, ρ_i is the resistivity caused by the scattering of the electron waves by static defects that disturb periodicity of the lattice. The expression $\rho = \rho_L + \rho_i$ is refered to as Mathiessen's rule. It states that ρ_L is independent of the number of defects at low concentrations while ρ_i is independent of temperature.

Residual resistivity $\rho_i(0)$ (at absolute zero) is the extrapolated resistivity at 0K since ρ_L vanishes at $T \rightarrow 0$.

The lattice resistivity at any temperature T, $\rho_L(T) = \rho - \rho_i(0)$ and is the same for different specimens of the metal although $\rho_i(0)$ may self vary widely.