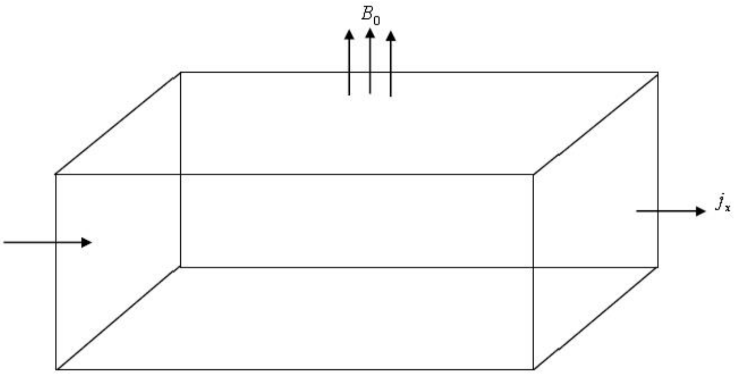


If current cannot flow out of the rod in the y direction, then drift velocity $\delta v_y = 0$ and thus from

$v_x = -\frac{e\tau}{m}E_x$ and $w_0\tau v_x = \frac{e\tau}{m}E_y$,

Then $E_y = -w_c\tau E_x = -\frac{eB_0\tau}{m}E_x$.

Recall that the current density is defined by $\frac{ne^2\tau}{m}E_x$.

$j = ne^2\tau\frac{E}{m}$.

Then, $j = ne^2\tau\frac{E_x}{m}$.

If we introduce a ratio $\frac{E_y}{j_xB_0} = -\frac{eB_0\tau}{m}.E_x.\frac{m}{B_0ne^2\tau E_x}$.

$\frac{E_y}{j_xB_0} = -\frac{1}{ne} \equiv R_H$.

where R_H is the hall coefficient which is negative for free electrons. The lower the carrier concentration the graeter the magnitude for the hall coefficient.

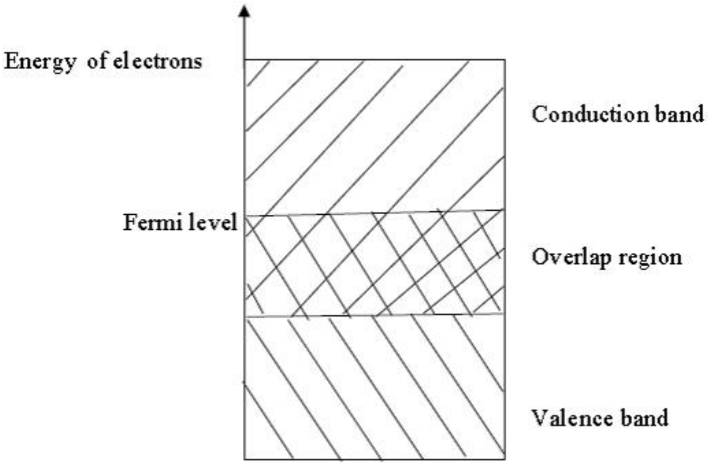
INTRODUCTION TO BAND THEORY OF SOLIDS

Electrons in a crystal are arranged in energy bands separated by regions of energy for which no wave-like electron orbitals exist. Such regions are called energy gaps or band gaps. We may visualise the difference between conductors, insulators and semi-conductors by plotting the available energies for the electron in the material.

An important parameter in the band theory is the fermi level which is the top of the available electron energy levels at low temperature. The position of the fermi level in relation to the conduction band is a crucial factor in determining electrical properties.

ENERGY BAND MODEL AT ABSOLUTE ZERO

A. CONDUCTORS:

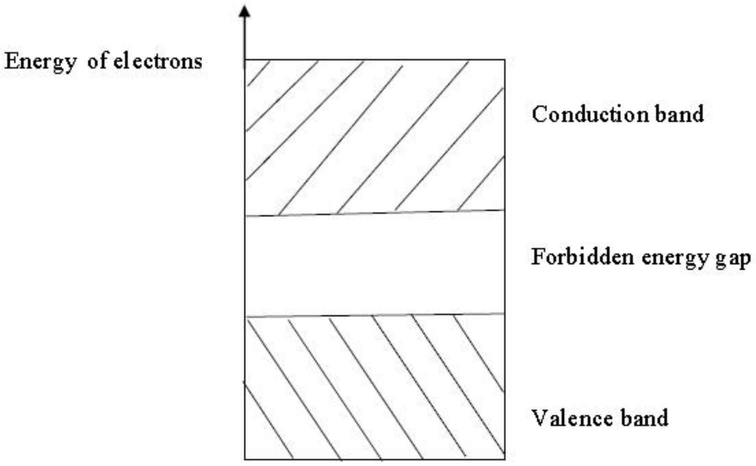


The valence band is the lowest energy level and is completely filled with electrons. Conduction band is the upper energy band and is empty of electrons since it corresponds to the un occupied higher levels in an isolated atom.

The forbidden energy gap is the region between the valence band and the conduction band. In metals or conductors the atomic and crystal structures are such that the valence band and the conduction band overlap hence the valence electrons move freely throughout the solid making metals excellent electric conductors.

B. INSULATORS:

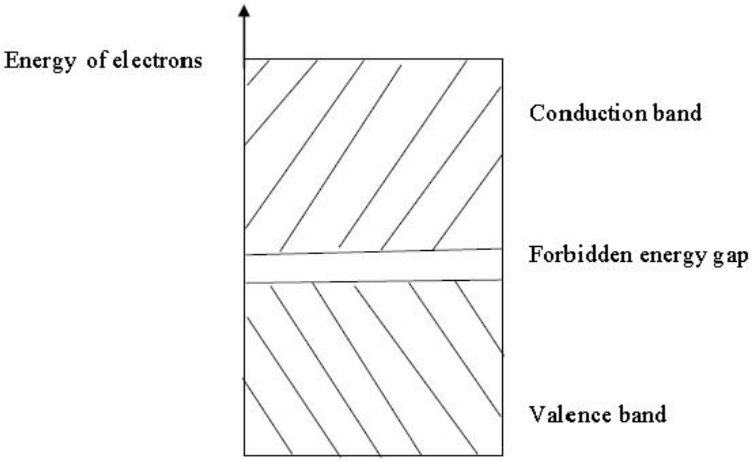
Insulators have a wide forbidden energy gap for electrons from the valence band to move



freely. As a result there are no electrons to act as carriers in the conduction band and therefore insulators cannot contribute to electrical conductivity.

Doping of insulators can drammatically change their optical properties but it is not enough to overcome the large band gap to make them good conductors of electricity.

C. SEMI CONDUCTORS:



In semi conductors, the forbidden energy gap is narrow, therefore at room temperature a few electrons can be excited from the valence band to the conduction band across this forbidden energy gap.

The corresponding electron vacancies in the valence band make it possible for the electrons in this band to contribute to electrical conductivity.

ORIGIN OF ENERGY GAP

Nearly free electron gap or theory:

The free electron theory is successful in explaining many of the properties of metals. However it doesnot explain why:

(i) some materials are metals and others insulators.

(ii) some metals have positive hall coefficients indicating the presence of mobile charged carriers within them.

Thus the nearly free electron theory tries to improve the free electron theory by taking into account the fact that the positive ions do not produce a uniform attractive potential but one with strong negative peaks at the lattice size i.e it includes a weak periodic perturbation meant to model the interaction between the conduction electrons and the ions in the crystal.

In dealing with the free electron gas in 3-D the wave function that satisfies the free particle Schrödinger equation and periodicity conditions takes the form:

$\psi_k(\vec{r}) = e^{i\vec{k}.\vec{r}}$.

Considering the time independent states represented by the two standing waves of the form

$e^{\pm\frac{i\pi x}{a}} = cos\frac{\pi x}{a} \pm isin\frac{\pi x}{a}$.

The standing waves are $\psi(+) = e^{\frac{i\pi x}{a}} + e^{-\frac{i\pi x}{a}} = 2cos\frac{i\pi x}{a}$.

$\psi(-) = e^{\frac{i\pi x}{a}} - e^{-\frac{i\pi x}{a}} = 2isin\frac{i\pi x}{a}$.

The ± signs indicate whether the standing waves change sign. The two waves pile up electrons at different values of potential energy in the field of ions of the lattice. This is the origin of the energy gap.

It is best described if the individual probability density, ρ of a particle is given by

$\rho = \psi\psi^* = |\psi|^2$

or $\rho = \psi^*\psi^{=}|\psi|^2$

Hence for a pure travelling wave ($\psi = e^{ikx}$), $\rho = e^{-ikx}.e^{ikx} = 1$.

This implies that the charge density is constant. For a linear combination of waves, the charge density is not constant.

Recall that $\psi(+) = 2cos\frac{\pi x}{a}$

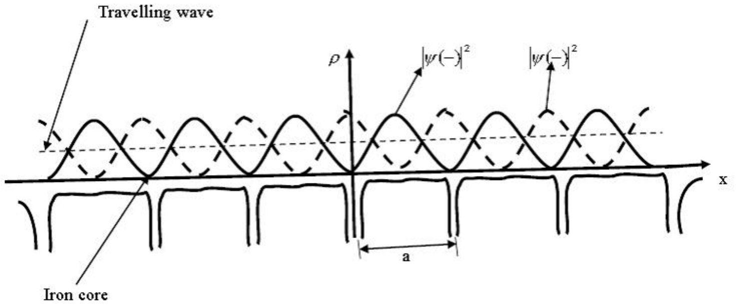
$\Rightarrow \rho_+ = 4cos^2(\frac{\pi x}{a})$

$\rho_+ \propto cos^2(\frac{\pi x}{a})$.

Similarly, for $\psi(-) = 2isin\frac{\pi x}{a}$

$\Rightarrow \rho_- = 4sin^2(\frac{\pi x}{a})$

$\rho_- \propto sin^2(\frac{\pi x}{a})$.



$\psi(+)$ piles up the electronic charge on the cores of the positive ions thereby lowering the potential energy in comparison with the average potential energy seen by a travelling wave. On the otherhand the wave function $\psi(-)$ piles up charge in the region between the ions thereby raising the potential energy in comparison with that seen by the travelling wave ($\psi(-)$ concentrates electrons away from the ion cores).

When the average values of the potential energy for the three charge distributions i.e for the travelling wave, $\psi(+)$ and $\psi(-)$ is calculated, the result is that the potential energy of ρ_+ is lower than that of the travelling wave whereas that of ρ_- is higher than that of the travelling wave.

$\rho_+ < \rho_{travellingwave} < \rho_-$.

Hence we have an energy gap of width E_g.

THE BLOCH THEOREM

It states that the solutions of the Schrödinger equation $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(r)}{\partial x^2} + V(x)\psi(r) = E\psi(r)$.

$$\frac{\partial^2\psi(r)}{\partial x^2} + \frac{2m}{\hbar^2}[E - V(r)]\psi(r) = o \qquad (*)$$

are the plane waves modulated by the function $U_k(\vec{r})$ which has the same periodicity as the lattice $\psi_k(\vec{r}) = e^{i\vec{k}.\vec{r}}U_k(\vec{r})$.

The potential $V(r)$ includes interaction of electrons with all the atoms in the solid as well as its interaction with other electrons. Thus the potential is periodic. Let the potential of the electrons satisfy $V(\vec{r}) = V(\vec{r} + \vec{R})$, where $\vec{R}$ is the lattice vector or period.

According to Bloch theorem the solution to the equation (*) will be given by $\psi_k(\vec{r}) = e^{i\vec{k}.\vec{r}}U_k(\vec{r})$ for a periodic potential U, where the function U_k has the same translation symmetry as the lattice or period i.e $U_k(\vec{r} + \vec{R}) = U_k(\vec{r})$.

Since the potential $V(\vec{r})$ is periodic then all observable quantities associated with the electron must be periodic. This also includes the quantity $|\psi(r)|^2$ which gives the electron probability. The only function which satisfies this requirement for all $\vec{r}$ is one of the exponential form i.e $e^{i\vec{k}.vecr}$. Thus the solution to the Schrödinger equation has the Bloch form.

Wave functions of the form $\psi_k(\vec{r}) = e^{i\vec{k}.\vec{r}}U_k(\vec{r})$ are called Bloch functions where the function $\psi_k(\vec{r})$ has the following properties:

1. It has the form of the travelling wave represented by $e^{i\vec{k}.vecr}$ which implies that the electron