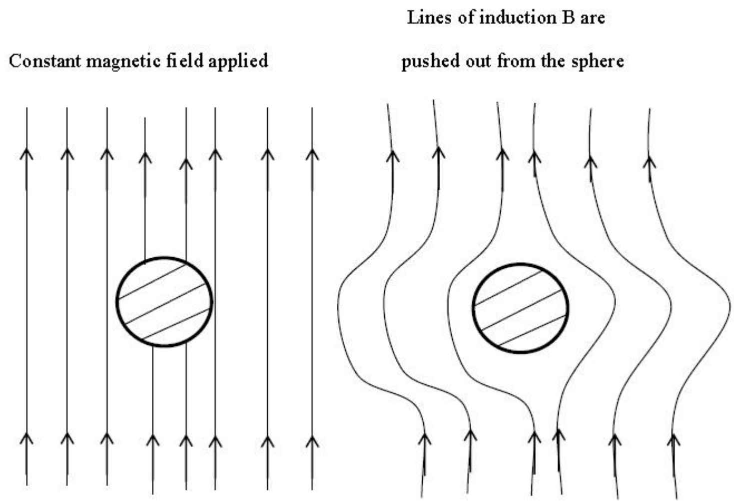
is referred to as the kelvin relation of the irreversible thermodynamics.

SUPERCONDUCTIVITY

This phenomenon refers to when electrical resistivity of a metal or alloy drops suddenly to zero when it is cooled to a low temperature. In otherwords at a critical temperature the specimen undergoes phase transition from a state of normal resistivity to a state of superconductivity. When in a superconducting state, electrical resistivity is zero or even close to zero but persisting electrical currents have been seen to flow without change in superconductivity rings.

Magnetic Properties of Superconductors

A bulk superconductor in a weak magnetic field will act as a perfect diamagnet with zero magnetic induction in the interior. When a specimen is put in a magnetic field and then cooled through the transition temperature for superconductivity the magnetic flux originally present is pushed out of the specimen.



The above effect is called Meissner effect. This is due to electric currents or screening current flowing on the surface of a superconductor in such a way as to generate a field equal and opposite to the applied field.

Consider a long thin specimen with a long axis which is parallel to the applied field B_a. The demagnetising field contribution will be neglected. Thus $B = B_a + \mu_0 M$ where M is magnetisation.

From $\vec{J} = \frac{\vec{E}}{\rho} = \sigma\vec{E}$.

If $\rho = 0$, while $\vec{j}$ is infinite, then $\vec{E}$ must equal to zero. But from Maxwell's equations, $\nabla \wedge \vec{E} = \frac{\partial \vec{B}}{\partial t}$.

If $\vec{E} = 0$, $\frac{\partial \vec{B}}{\partial t} = 0$, But $\vec{B} \neq 0$.

This shows that the flux through the metal cannot change on cooling through the transition process i.e the Meissner effect contradicts $\vec{B} \neq 0$.

THE LONDON EQUATION

We recall that the electrical conduction in a normal state of a metal can be described by Ohm's law given as

$$\vec{j} = \sigma\vec{E} \tag{1}$$

The above equation needs to be modified drastically to describe conduction amd Meissner effect in the superconducting states. Assume that in the superconducting state, the current density is proportional to the vector potential $\vec{A}$.

Then $\vec{j} \propto \vec{A}$. i.e $\vec{j}$= (constant)$\vec{A}$.

This constant takes the form of $\frac{-1}{\mu_0\lambda_l^2}$, where λ_l is a constant with dimensions of l. Thus

$$J = \frac{-1}{\mu_0\lambda_l^2} \tag{2}$$

which is the London equation and λ_l is called the London penetration depth given by

$$\lambda_l^2 = \frac{\epsilon_0 mc^2}{ne^2} \tag{3}$$

where m is the mass of the particle, c is the speed of light, n is the number of particles per unit volume.

If we use vector calculus, taking the curl of equation (2) gives

$\nabla \wedge \vec{j} = \frac{-1}{\mu_0 \lambda_l^2} \nabla \wedge \vec{A}$

But $\nabla \wedge \vec{A} = \vec{B}$

$$\nabla \wedge \vec{j} = \frac{-1}{\mu_0 \lambda_l^2} \vec{B} \tag{4}$$

Equation (4) is another way of expressing the London equation.

From Maxwell's equation

$$\nabla \wedge \vec{B} = \mu_0 \vec{j} \tag{5}$$

Taking the curl of equation (5) we have

$$\nabla \wedge (\nabla \wedge \vec{B}) = \mu_0 \nabla \wedge \vec{j} \tag{6}$$

But $\nabla \wedge (\nabla \wedge \vec{B}) = \nabla(\nabla . \vec{B}) - \nabla^2 \vec{B}$

$$\nabla \wedge (\nabla \wedge \vec{B}) = -\nabla^2 \vec{B} \tag{7}$$

since $\nabla . \vec{B} = 0$.

Then equation (6) and (7) gives,

$$\mu_0 \nabla \wedge \vec{j} = -\nabla^2 \vec{B} \tag{8}$$

Substituting for $\nabla \wedge \vec{j}$ using equation (4)

$\mu_0 (\frac{-1}{\mu_0 \lambda_l^2} \vec{B}) = -\nabla^2 \vec{B}$.

$\Rightarrow \nabla^2 \vec{B} = \frac{\vec{B}}{\mu_0 \lambda_l^2} . \mu_0$

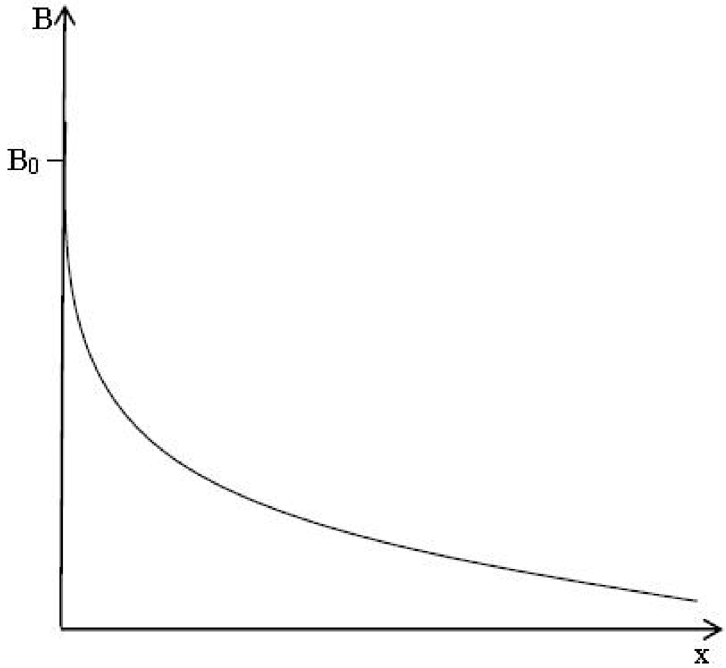
$\nabla^2 \vec{B} = \frac{\vec{B}}{\lambda_l^2}$

$$\lambda_l^2 \nabla^2 \vec{B} - \vec{B} = 0 \tag{9}$$

The solution to equation (9) is of the form

$$B(x) = B_0 exp(\frac{-x}{\lambda_l}) \tag{10}$$

where x is the distance inside the conductor.



Thus the London equation predicts exponential decay of magnetic field into a superconductor occupying the region $x > 0$.

COHERENCE LENGTH, (ϵ)

This is the measure of the distance within which the superconducting electron concentration cannot change drastically to the partially varying magnetic field.

Consider a plane wave

$$\psi(x) = e^{ikx} \tag{1}$$

and a strongly modulated wave function

$$\varphi(x) = \frac{1}{\sqrt{2}}(e^{i(k+q)x} + e^{ikx}) \tag{2}$$